ANALYSIS

Edited by A. E. Duncan-Jones with the cooperation
of L. S. Stebbing C. A. Mace and G. Ryle

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ON THE THEORY OF AXIOM-SYSTEMS

By OLAF HELMER

(1) *Axiom-systems*

IN this paper I want to deal with some problems concerning the theory of axiomatics, and to give a short account of the present state of this theory without any claims for completeness.

An *axiom-system* (AS) of a theory is a (usually finite) set of theorems belonging to that theory, with the property of enabling us to deduce from them together with purely logical formulae all the remaining theorems of the theory according to the rules of logical inference. Every AS contains a set of *undefined fundamental concepts* ; these may be denoted by $g_1, g_2, \ldots, g_r$. The axioms may be called $A_1, A_2, \ldots, A_n$; if it is necessary to emphasize their containing the concepts $g_1, g_2, \ldots, g_r$ we shall write $A_1(g_1, g_2, \ldots, g_r), \ldots, A_n(g_1, g_2, \ldots, g_r)$ instead. The logical product of the axioms may be A or more explicitly $A(g_1, g_2, \ldots, g_r)$.

(2) *The class of statements a theory is concerned with.*

Let P be the set of all statements being significant with respect to the theory under consideration, i.e. the class of all those statements which are expressible in terms of that theory ; P always contains the class of all purely logical propositions. In P the following five sub-classes may be distinguished : (1) the class T of tautologies (deducible without using A), (2) the class D of deducible statements (by means of A), (3) the class U of " undecidable " statements, (4) the class N of disprovable statements

(negations of the elements of D), and (5) the class C of contradictions (negations of tautologies). T and C are contained in D and N respectively. The statements belonging to D–T are usually called " theorems."

It must, however, be noticed that the usual formulae of the propositional calculus do not necessarily hold for all statements of P. If, for instance, p belongs to U, we have neither p nor $-p$ in this theory, which means that p does not satisfy the tautology $p \vee -p$. This remark, which has specially been made by the so-called intuitionists, is frequently misleadingly said to be a denial of the " Principle of the excluded middle."

The reason why the logistic formulae fail with regard to the statements of U is as follows : The statements p of the considered theory are not really propositions but propositional functions $p(g_1, g_2, \ldots, g_r)$ of the fundamental concepts. Hence " p is deducible from A " does not mean that the so-called material implication $A \supset p$ holds, but it involves the general implication $(g_1, g_2, \ldots, g_r).A(g_1, g_2, \ldots, g_r) \supset p(g_1, g_2, \ldots, g_r)$. Similarly, " $-p$ is deducible from A " means : $(g_1, g_2, \ldots, g_r).A(g_1, g_2, \ldots, g_r) \supset -p(g_1, g_2, \ldots, g_r)$, and now it is obvious that sometimes neither of these formulae need be fulfilled. The assertion that in this case the Principle of the excluded middle is violated has therefore to be taken with a grain of salt. (If deducibility only meant material implication, U would always be empty, since any p would be decidable because of the tautology $A \supset p . \vee . A \supset -p$.)

(3) *Two examples of axiom-systems.*

To illustrate the following considerations we may introduce some examples of AS's.

Peano's famous AS for the non-negative integers consists of 5 axioms containing 3 undefined concepts g_1, g_2, g_3 which I shall denote in accordance with their usual interpretation by z (" zero "), N (class of non-negative whole " numbers "), and S (" successor ") :

A_1 : $z \in N$

A_2 : $(x).x \in N \supset S'x \in N$

A_3 : $(x,y) : x \in N.y \in N.S'x = S'y. \supset .x = y$

A_4 : $(x).x \in N \supset S'x \neq z$

A_5 : $(A) : .z \in A : (x).x \in A \supset S'x \in A : \supset .N \subset A$

(A_3 is the axiom of infinity, A_5 that of mathematical induction.)

Hilbert's AS of Euclidean geometry contains 7 undefined concepts which may be denoted by P (class of " points "), S (class of " straight lines "), Pl (class of " planes "), I (relation of " incidence "), B (relation of " betweenness "), Cs (" congruence of segments of straight lines "), Ca (" congruence of angles "). A few examples of the great number of axioms will be sufficient :

" For any two different elements of P there is at least one element of S standing to both of them in the relation I."

Axiom of parallels : " For any element s of S and any element p of P not standing in I to s, there is at most one element s' of S standing to p but not to s in the relation I."

Hilbert's last axiom is the so-called

Axiom of completeness : " The points, straight lines and planes, related by I, B, Cs and Ca, form a system not being capable of any extension without violating one of the preceding axioms."

The admissibility of this axiom is open to certain objections because of its reference to the preceding axioms, but we are not here entering into a detailed discussion of this problem[1].

(4) *Interpretations of an axiom-system.*

If, with the fundamental concepts $g_1, g_2, \ldots, g_r$ of the AS $A_1, A_2, \ldots, A_n$, certain objects are correlated by means of " correlative definitions" (*Zuordnungsdefinitionen*), and if all the axioms are fulfilled in that system of objects, this system is called an interpretation of the AS.

For instance, for Hilbert's geometrical AS a physical interpretation can be given in which straight lines are interpreted as light-rays ; another interpretation of the same system is an arithmetical one, the straight lines being interpreted as linear equations. Similarly, the cardinal numbers introduced by Whitehead and Russell as certain classes of classes[2], form a logical interpretation of Peano's AS. Further, any arithmetical sequence can be conceived as an arithmetical interpretation of the same AS.

[1] Compare : *D. Hilbert*, Grundlagen der Geometrie, 7th ed. 1930, and : *O. Helmer*, Axiomatischer Aufbau der Geometrie in formalisierter Darstellung, Schriften des Mathematischen Seminars Berlin, vol. 2, 1935.

[2] *A. N. Whitehead* and *B. Russell*, Principia Mathematica.

(5) *Consistency and definiteness.*

If $A_1, A_2, \ldots, A_n$ is a given AS the well-known problems arise as to whether this system is *consistent* and *definite* (instead of 'definite' sometimes the terms 'categorical' or 'complete' are used; the German word is 'entscheidungsdefinit'). Consistency means that no element of C belongs to D; (in case a single element of C belongs to D, any statement is deducible, i.e. D = P; for every contradiction implies any other statement). Definiteness means that U is empty, i.e. that any significant statement is either deducible or disprovable.

An AS is called *satisfiable* if there exists at least one interpretation. Satisfiability certainly implies consistency. Conversely, an AS formally written down can, as long as it is consistent, always be conceived as its own interpretation; hence 'satisfiable' and 'consistent' are equivalent properties.

An AS is called *monomorphic* if any two interpretations are isomorphic, i.e. if they agree in all those of their properties with which the theory in question is concerned. Further, if there is a statement with the property that both it and its negation can be consistently added to the given AS, this AS is called "*forkable*" (at this statement); such a statement must obviously belong to U. If one adds it or its negation to the original AS one gets two consistent systems the interpretations of which are no longer isomorphic. From this we see that 'definite', 'monomorphic' and 'non-forkable' are equivalent properties. (Completeness in the sense of definiteness has to be strictly distinguished from that sort of completeness which is postulated in Hilbert's completeness-axiom.)

In practice, the performance of a consistency- or definiteness-proof has to face considerable difficulties.[3] One usually tries to give such a proof by referring to a given interpretation of the AS. If in particular that interpretation is a physical one, even in the most favourable case one can only obtain statements which have—like all statements founded on experience—not more than a certain probability. An arithmetical or logical interpretation, on the other hand, may yield a reduction of the problems in question to the solution of the corresponding ones in arithmetic or logic. By this method, Hilbert has been able to show that

[3] See for instance: *K. Gödel*, Über formal unentscheidbare Sätze der "Principia Mathematica" und verwandter Systeme, Monatshefte für Math. u. Physik 38, 1931.

his system of Euclidean geometry is consistent and definite if the same is the case with respect to arithmetic.

(6) *Independence.*

An AS $A_1, A_2, \ldots, A_n$ is called *independent*, if each of the axioms is undecidable with respect to the remainder.

If A is the conjunction of the axioms, 'consistent' and 'definite' can be interpreted as properties of A because the splitting up into $A_1, A_2 \ldots, A_n$ is irrelevant. 'Independent' on the contrary is a property of $A_1, A_2, \ldots, A_n$; for if one splits A up into a different conjunction of axioms—say $B_1, B_2, \ldots, B_m$—the question whether this system is independent is no longer the same.

One usually postulates independence for economical reasons: one wishes to get along with as few axioms as possible and to leave out such as follow from the remainder. In principle, one could equally well build up a theory upon a dependent AS, and from this point of view, the demanding of independence does not seem to be very important. Besides, an AS does not satisfy a very strong condition if it is really independent. For independence can always be achieved in a trivial way, namely by taking the logical product A as the only axiom without splitting it up; a single axiom certainly represents an independent system.

But—as mentioned before—the postulate of independence is meant to be an economical principle. In order to satisfy it in accordance with its meaning, one would first have to split A up into as many axioms as possible (i.e. into axioms with as little content as possible) and then to demand independence from them. But whether such a representation of A as a conjunction of axioms with as little content as possible is always and uniquely achievable, is, up till now, only known in the case of the elementary calculus of propositions (where any proposition can be transformed into the so-called conjunctive standard-form). By means of an example one can see that it is not even plausible that a conjunctive standard-form of the desired form can be obtained at all in the general case: Let us imagine an AS containing the undefined concept g; one of the axioms may be:

"There is one and only one x such that $g(x)$."

This axiom can trivially be split up into two axioms, since ''one

and only one" is replaceable by the conjunction "at least one and at most one." But we can also get a sub-division into 5 axioms :

(1) "The number of x such that $g(x)$ is odd."
(2) "The number of x such that $g(x)$ is less than 8."
(3) "The number of x such that $g(x)$ is not equal to 7."
(4) "The number of x such that $g(x)$ is not equal to 5."
(5) "The number of x such that $g(x)$ is not equal to 3."

By the same principle, the given axiom can obviously be split up into any number n of independent axioms.

We learn from this that the property of an AS of being independent is not strong enough to be of any importance. In consequence, one frequently replaces the postulate of independence by a stronger one. One may for example, with E. H. Moore, call an AS *completely independent*, if any conjunction of axioms of this system is compatible with the conjunction of the negations of the remaining axioms. That this condition is stronger than the ordinary independence can be seen from the following example : The integer n may be characterized by these axioms :

A_1 : "n is a prime number."
A_2 : "n is less than 11."
A_3 : "n is one of the numbers 2, 3, 9, 10, 11, 12."
A_4 : "n is a divisor of 770."

These axioms are consistent and definite since "2" and only "2" satisfies them. Further the AS is independent as no contradiction arises by replacing one axiom after the other by its negation :

$-A_1.A_2.A_3.A_4$ is satisfied by 10,
$A_1.-A_2.A_3.A_4$ is satisfied by 11,
$A_1.A_2.-A_3.A_4$ is satisfied by 5,
$A_1.A_2.A_3.-A_4$ is satisfied by 3.

But the system is not completely independent because no number can be found which satisfies $A_1.A_2.-A_3.-A_4$.

Different methods of strengthening the condition of independence have been attempted by H. M. Sheffer and A. Church who have introduced the notions of *mutual primeness* and *irredundancy* respectively. (Two statements are called mutually prime, if they have no non-tautological consequence in common ;

an AS is then called mutually prime, if, for each axiom, this and the logical product of the remaining axioms are mutually prime. An AS is called irredundant, if none of its axioms can be replaced by some genuine part of it such that the AS obtained in this way is equivalent to the original one.)

It is worth mentioning that occasionally independence investigations are undertaken not pursuing economical aims. The most famous example is that of the axiom of parallels whose independence of the remaining geometrical axioms caused the introduction of the so-called non-Euclidean geometries. The fact that the axiom of parallels is independent of the other axioms, can also be expressed in this way: The geometrical axioms without the axiom of parallels form an AS which is not definite, but forkable at the postulate of parallels. Similarly, the independence proof of Archimedes' axiom of geometry is equivalent to establishing a non-Archimedean interpretation of geometry.

(7) *Adequate axiomatization of a given theory.*

If one is setting up an AS one generally only reconstructs axiomatically an already existing theory. Then, apart from the independence proof of the system, another task arises akin to but not identical with it, namely to show that the theory founded upon the chosen axioms is complete in the sense of really agreeing with that theory whose reconstruction was intended, i.e. that it actually contains every concept and statement required. This can be demonstrated firstly by showing that all concepts of the theory in question can be constituted from the chosen undefined concepts of the AS by means of formal definitions, and secondly that all statements of the theory are deducible from the axioms by means of formal logical inference. In practice it will always suffice to give these proofs only for those concepts and theorems which are comparatively fundamental.

(8) *Some remarks about the theory of types.*

From the axiomatic point of view, I will concern myself in this paper only shortly with the theory of logical types. In the usual way one can perform a hierarchical sub-division of the concepts of any scientific theory. Starting with a range V of

individuals (the fundamental type) one may introduce one- or many-termed predicates of these individuals, further predicates of these predicates, and so on. To each of these types of propositional functions there is a corresponding type of the extensions (classes or relations) belonging to those predicates. (According to recent investigations[4] the formal distinction between propositional functions and the correlated relations is redundant ; the same would then hold for the corresponding types.) While, on the one hand, the logical types supply a hierarchy of all concepts of a theory, they represent, on the other, the variability ranges for the variables occurring in the theory.

Each concept constitutable from the undefined concepts of the given AS by means of definitions, belongs to one and only one of the types introduced above (the undefined concepts themselves nearly always belong—although there is no theoretical necessity for it—to the types of the zero or the first level, i.e. to V or to the propositional functions of individuals). From this we know which elements must at least be contained in the logical types (namely the fundamental and all formally definable concepts). But the problem whether the types contain any further elements and, if so, which ones, has been solved satisfactorily neither by Whitehead and Russell in " Principia Mathematica " nor by Ramsey in " The foundation of mathematics "[5] nor—as far as I can see—by L. Chwistek in " Theory of constructive types."[6]

This problem seems to me to be of some importance because of the part it plays in the interpretation of an AS. For in my opinion an interpretation of an AS is completely determined only if the variability ranges of the occurring variables are precisely characterized, and these are—as has been said before—the logical types. If this is not the case, the sense of those theorems whose formal representation contains apparent variables, does not seem to be entirely determinate.

(9) *Axiomatic of axiomatics.*

The theory of AS's is—like any theory—capable of being formalized and axiomatized itself. In this direction proceed,

[4] See : *R. Carnap*, Logische Syntax der Sprache, Vienna 1934.
[5] Proceedings London Math. Soc., series 2, vol. 25, part 5.
[6] Cracow 1923.

for instance, the investigations of H. Scholz and F. Bachmann in Münster (Westfalia)[7] and of A. Tarski and J. Lukasiewicz in Warsaw.[8]

It may be of some interest to give at least a superficial impression of Tarski's system. He chooses as his fundamental concepts S (the class of all statements) and the function F(X) (the class of all consequences of X, where X is a sub-class of S). Some of his axioms are the following :

A_1 : $\mathrm{Nc'S} \leq \alpha_0$ [9]

A_2 : $X \subset S . \supset . X \subset F(X) \subset S$

A_3 : $X \subset S . \supset . F(F(X)) = F(X)$

A_4 : $X \subset S . \supset . F(X) = s'F''\hat{Y}(Y \subset X . \mathrm{Nc'Y} < \alpha_0)$

Further, some of Tarski's definitions may be mentioned :

D_1 : $X \mathbf{eq} Y \quad =\mathrm{Df} \quad X \cup Y \subset S . F(X) = F(Y)$

(" logically equivalent classes of statements "),

D_2 : $X \in \mathbf{W} \quad =\mathrm{Df} \quad X \subset S . -(X \mathbf{eq} S)$

(" consistent class of statements "),

D_3 : $X \in \mathbf{V} \quad =\mathrm{Df} \quad X \subset S : . (Y) : Y \in \mathbf{W} . X \subset Y . \supset . X \mathbf{eq} Y$

(" definite class of statements "),

D_4 : $X \in \mathbf{U} \quad =\mathrm{Df} \quad X \subset S : (Y) : Y \subset X . Y \mathbf{eq} X . \supset . Y = X$

(" independent class of statements "),

D_5 : $Y \in \mathbf{B}\,(X) \quad =\mathrm{Df} \quad X \mathbf{eq} Y . Y \in \mathbf{U}$

(" basis of a class of statements "),

D_6 : $Y \in \mathbf{Ax}\,(X) \quad =\mathrm{Df} \quad X \mathbf{eq} Y . \mathrm{Nc'Y} < \alpha_0$

(" AS of a class of statements ").

An important theorem provable in Tarski's system is the following :

$$X \subset S . \supset . \mathbf{B}(X) \neq O$$

If one wants to interpret Tarski's system, one has to say what S and F are meant to be, i.e. one has to choose—in Carnap's terminology—formation-rules (to determine S) and transformation-rules (to determine F). For problems like that, Carnap's " Syntax " (see footnote 4) is competent.

[7] See : Semester-Berichte Math. Seminar Münster, especially Heft 6, 1934-35.
[8] See : Comptes rendus, société des sciences, Varsovie, Classe III, 1930.
[9] Alpha is here used in place of Aleph as a symbol for transfinite numbers.

(10) *The problem of simplicity of an axiom-system.*

An essentially unsolved problem is the question how to characterize formally the "simplicity" of an AS. Which, amongst the numerous possible AS's of a theory, is the simplest one? Attempts to approach the solution are usually undertaken in two directions: Either one refers to the number of axioms, or to the number of undefined concepts involved in the AS. In the first case we are led to independence investigations; their questionability has already been discussed under (6). In the second case it is called a simplification of the system if a reduction of the number of undefined fundamental concepts is possible. Such successes, however, must be received with the greatest caution, and one must not congratulate oneself too quickly when one has succeeded in replacing the r fundamental concepts $g_1, g_2, \ldots, g_r$ of an AS by $r-1$ fundamental concepts $h_1, h_2, \ldots, h_{r-1}$ from which the former ones are definable. For we shall prove now that a reduction to only one fundamental concept can always be achieved.

For this purpose let us introduce the following notations: If $R(x_1, \ldots, x_r)$ is an r-termed relation, we shall denote its r domains by $D_1, \ldots, D_r$:

$$D_1'R =Df \hat{x}_1[(\exists x_2, \ldots, x_r).R(x_1, x_2, \ldots, x_r)]$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot$$

$$D_r'R =Df \hat{x}_r[(\exists x_1, \ldots, x_{r-1}).R(x_1, \ldots, x_{r-1}, x_r)]$$

Further, we shall generalize the Principia-definition

$$x \downarrow y =Df \hat{u}\hat{v}(u=x.\ v=y)$$

in this way:

$$\downarrow_r(x_1, x_2, \ldots, x_r) =Df \hat{u}_1\hat{u}_2 \ldots \hat{u}_r(u_1=x_1.u_2=x_2 \ldots u_r=x_r)$$

If now an AS is given which contains the fundamental concepts $g_1, g_2, \ldots, g_r$, one may introduce the one fundamental concept

$$g =Df \downarrow_r(g_1, g_2, \ldots, g_r)'$$

instead; for each of the concepts $g_1, g_2, \ldots, g_r$ is expressible in terms of g alone:

$$g_\rho =Df \breve{\iota}'D_\rho'g \qquad (\rho = 1, 2, \ldots, r)$$

(where as usual $\iota'x = [x]$ and $x\breve{R}y = yRx$). Hence the original AS $A(g_1, g_2, \ldots, g_r)$ can be replaced by

$$A'(g) =Df A(\breve{\iota}'D_1'g, \breve{\iota}'D_2'g, \ldots, \breve{\iota}'D_r'g)$$

We may learn from this, that a reduction to a smaller number of undefined concepts does not necessarily mean a genuine simplification ; on the contrary, the problem seems to arise : under what conditions is a given concept reducible to several, in some sense simpler, concepts ? As to axioms, such a reduction may be attempted by means of splitting up into a logical product of weaker axioms ; but an analogue to that does not exist for concepts, which makes the situation comparatively more complicated.

Epsom Downs, June 1935.

THE ANALYSIS OF SENSE DATA

By G. A. PAUL

PHILOSOPHERS sometimes carry on an activity which they call " trying to find the analysis of sense-data." It is generally difficult to understand their accounts of what they are doing and what they hope to gain if they succeed. Mr. J. O. Wisdom's attempt in *Analysis* (no. 10, pp. 78–80) is no exception, and since his argument is typical of many it may be worth while trying to find out what he is actually doing when he says he is looking for the analysis of sense-data, and what conclusions he draws from the analysis he claims to find.

We understand to some extent (and believe that experts understand to a greater extent) what a chemist is going to do when he says he is going to analyse a certain compound, or a historian when he says he is going to analyse a given historical situation, or a psycho-analyst a patient : but we do not similarly understand what a philosopher intends to do when he says he is going to analyse selves, or propositions, or sense-data, or anything else, and so we expect some explanation. But Mr. Wisdom gives us none, nor does he give *other instances* of the sort of thing he is going to do, nor even some *criterion* by which we shall know whether the analysis has been found.

He does, however, tell us something that " must " be done *if* we are to find the analysis of sense-data, viz., " some particular must be found " (78, 11)[1]. By a " particular " he means something which " cannot " characterise anything, i.e. presumably, something which it is " logically impossible " should characterise anything (78, 9).

The shape of a sense-datum is not a particular because it makes sense and is not self-contradictory to say of the shape of a sense-datum that something is characterised by it ; and for a similar reason the colour of a sense-datum is not a particular. " It is, therefore," he says, " worth enquiring whether the here or the now can be particulars." (78, 12)

[1] The first figure in each bracket is the page number of *Analysis* (no. 10) ; the second is the number of the line of Mr. Wisdom's article on that page.

Not all the phrases in inverted commas are quotations from Mr. Wisdom's article. Some are merely philosophical phrases which I felt compelled to use, but which appear to me especially barbarous.

The " enquiry " proceeds on the following lines :—Suppose we use the word " here " " to refer to an absolutely specific place in visual space " and the word " now " to refer to an absolutely specific time " as consciously apprehended " (78, 8), then (he says) " if ' I am seeing a sense-datum ' were translated by ' A colour and a shape are here now,' something would have been omitted, namely the *particularity* of the sense-datum." (79, 1) I shall try to say later what I think he has in mind when he says this, but at the moment I only wish to draw attention to the curious recipe for supplying the missing " particularity " :—The recipe is to " translate " " I am seeing a sense-datum " into a sentence in which the words " here " and " now " occur *as nouns* only (and of course still refer to absolutely specific places and times), and to write them with capitals to remind us that they are nouns and not adverbs as before. (79, 11) The proposed sentence is " Now is here characterised by a colour and by a shape " (or indifferently " Here is now characterised by a colour and by a shape "). But this, it is held, is unsatisfactory because in the first formulation the word " here " is an adverb and in the second the word " now " is an adverb, and so we might still be misled into thinking that Here " need not " *now* be so characterised and that Now need not *here* be so characterised, whereas it follows (Mr. Wisdom says) from his usage of the words " Here " and " Now " that Here *must* be now and Now *must* be here (i.e., presumably, that it is *logically impossible* that Here should not be now, etc. . . .). (79, 19)

But there is in fact nothing that Mr. Wisdom has said about his use of the words " Here " and " Now " or any example he has given of their use from which it *follows* that Here must be now and Now must be here. The fact simply is that Mr. Wisdom *wishes* to use the words in this way, and in order not to forget that this is the way they are to be used we have to " resort " to combining the two into a single word and saying " I am seeing a sense-datum " means " Herenow is characterised by a colour and shape." It appears (1) that in accordance with instructions " some particular has been found " ; for we are told that " Herenows seem to be particulars as required " (79, 27). And (2) that this *is* " the analysis of sense-data " (79, 26).

It will now be convenient to examine the argument up to this point :—From the last paragraph of his paper it appears that

Mr. Wisdom would say the purpose of his analysis was to gain some sort of " insight " into something or other, the nature or structure of sense-data, or " the form of propositions " of a certain class (80, 32). We apparently gain insight into " the nature of sense-data " when we see that it is misleading to talk of the occurrence of a sense-datum at a certain place or time and that the misleadingness is removed when we talk, instead, of attributing characteristics to a Herenow. (80, 30–4).
And being thus misled as to the real nature or " structure " of sense-data leads us to deny or to ignore certain truths about them (e.g., such as are expressed in Mr. Wisdom's three "conclusions " which are mentioned below).

He therefore begins his attempt to discover what is the real structure of sense-data by trying to find the " particular " which is, as it were, the core of the sense-datum. What he seems to have in mind when he says that if we say " A colour and a shape are here now " the " particularity " of the sense-datum has been omitted is that neither the colour nor the shape nor the place nor the time we mention " need be " peculiar to *this* sense-datum. I.e., it is logically possible that *other* sense-data have occurred in the *same* place as this one, that *other* sense-data are occurring at the *same* time, that *other* sense-data have the *same* colour and the *same* shape. I.e., nothing has been mentioned which it is not logically possible that this sense-datum should have in common with others. A sense-datum seems to be wholly made up of elements which are common property : we want to know " what it is that makes it peculiarly itself as distinct from all others ? " It is not difficult to manufacture an " answer " to such a " question " :—Two sense-data, it is noted, may occur at the same time, succeed each other in the same place, and have the same colour and shape ; but two sense-data can't occur *in the same place at the same time*. So if we coalesce the time and the place into a single entity, we have " found " something which is peculiar to the sense-datum in which it is an element.

This element is dubbed a " Herenow." That a Herenow satisfies Mr. Wisdom's explicit definition is obvious. For to say of anything that it " is characterised by being Herenow " or " is Herenow " would not be using the word " Herenow " as a noun, and therefore would not be to use it in the way stipulated by Mr. Wisdom. But it appears doubtful (owing to the am-

biguity of " is characterised by ") whether it really satisfies the definition he has in mind. For there can be little doubt that he has in mind the familiar jingle about particulars which adds that a particular is a thing which *can have* characters. Now the sense in which a Herenow could be said to have characters is different from one in which a colour or a shape or anything else is ordinarily said to have characters. (And it was probably some more ordinary sense that Mr. Wisdom had in mind in writing his second paragraph). For part of what is meant when it is said that a Herenow has characters is that a time has characters ; and if it means anything to say that a time has characters (in the way in which Mr. Wisdom tries to do here), it probably means simply that something has some character (in some ordinary sense) *throughout* that time. Since it is unlikely that Mr. Wisdom had this sense in mind when he began to look for something which could have characters and yet not be a character, it appears that he has not found anything to answer to the description of what he was looking for.

But whether or not there is a thing called a Herenow which answers the description, it seems clear from the fact that " is characterised by " is being used in this curious way, that the formulation " Herenow is characterised by a colour and a shape," which is supposed to give insight into the structure of sense-data and help us to avoid pitfalls, so far from in some way explaining the meaning of a sentence which contains the phrase " . . . is characterised here and now " requires to have its meaning explained in terms of some such sentence.

Having " found the analysis of sense-data " Mr. Wisdom goes on to point out certain " peculiarities " (79, 33) of Herenows, viz., (1) Herenows " can have a duration corresponding to as long or as short a period (of clock-time) as we please " (80, 1). (2) " It is only when some interruption occurs that one Herenow succeeds another " (80, 3). These two " peculiarities " are produced as if they were things that had been discovered by examining this curious sort of object, a Herenow, and observing its properties. Whereas examination of Mr. Wisdom's procedure makes it clear that the " observing " of these surprising properties is nothing but arbitrarily choosing to use the word " Herenow " in certain ways, which are no more and perhaps *less* convenient than some other ways in which he might

have chosen to use it. E.g., suppose I see a red after-image for five minutes, then if I follow his rules I may express that fact (to myself) by saying " Herenow is red " (provided there is no " interruption ") : but he might equally well have chosen to say that Herenows are but fleeting, " momentary " things, in which case I might express the same thing by some such phrase as " An infinite succession of Herenows are red." That the terminology is arbitrary is best shown by asking what it would be like for it to be the case that Herenows can *not* have more than momentary existence. At first it looks as if it were being said that our immediate visual experience is made up of mere fleeting flashes : but clearly this is not what is being said, because even if it were it would still be true that they *could* (i.e., that it is logically possibly that they should) endure throughout a considerable period. What it *would* be like for it to be the case that Herenows can have only momentary existence would be for it to be part of the instructions for the use of the word " Herenow " that it is to be used only for things having a momentary existence. If our experiences changed in such a way that the fleeting flashes became long-drawn-out, we should not say that the logically impossible had occurred but should say that what we now saw were infinite successions of Herenows (or some such phrase) or should cease to call them Herenows and should say that we no longer saw Herenows but only (say) Herethroughouts. Thus, to say as Mr. Wisdom does that Herenows *can* have a duration is not to make an empirical observation about some object which we see in immediate experience but is to say something about the terminology he is going to use to describe such empirical observations, viz., that, no matter how long a thing lasts in immediate experience, that alone is not going to prevent it being called a Herenow.

In the case of the second " peculiarity " it is even clearer that it is just a way of expressing a convention which tells us under what conditions it is to be said in this new terminology that one Herenow has succeeded another. The convention might have been for example that, instead of saying that one Herenow succeeds another whenever *any* " interruption " occurs, we were to say that one succeeds another only when *some specific sort* of " interruption " occurs, e.g. a sneeze (in a purely sensory sense).

We must consider lastly the " interesting results " : (1) " A sense-datum can be numerically the same for a considerable time, and will be changed only by a change of Herenow " (80, 8).

(This does not, as Mr. Wisdom claims, follow from what he has already said. It assumes a convention connecting the use of the word " sense-datum " with the use of the word " Herenow," which may be summed up in the phrase " One Herenow, one sense-datum".)

(2) One sense-datum " can be changed into an exactly similar sense-datum by a suitable change of Herenow " (80, 10).

("Identity in change" was known to the ancients; it has been left till now to discover " change in identity.")

Mr. Wisdom speaks as if these conclusions were arrived at by examining the contents of our visual field and finding out their " real nature." But there is nothing in the nature of sense-data which by itself determines under what conditions it is correct to say they are numerically the same or different. Some *convention* requires to be adopted to the effect that under such and such conditions we are to say " The same sense-datum appeared twice." " This sense-datum is numerically different from that one", or that such phrases are not to be used at all in talking about sense-data. It may happen that a philosopher consciously decides to use the word in some new way, or that without consciously deciding to do so he adopts some usage which has not occurred before, or that, consciously or unconsciously, he follows some usage which has already been adopted elsewhere. Some parts of the usage Mr. Wisdom adopts have undoubtedly been adopted before ; but in some other respects it is a little surprising. And since he believes that his is " the correct usage " as being " founded in the nature of things " he is led to believe that he has made some surprising discovery about sense-data.

Let us examine the usage he adopts :—Suppose I look at a light and then look away and have a red after-image which lasts ten seconds. On his usage (granted there is no " interruption ") I can express that by saying (to myself) " Herenow is red," and (presumably) to others by saying " I have been seeing the *same* red sense-datum for ten seconds". (He claims that this is better than a use in which sense-data are defined as " momentary " in

which we would need to say something like " During the last ten seconds I have been seeing an infinite succession of sense-data all the same shade of red ".) But suppose that while I am seeing the after-image I am aware of a clock ticking, then on Mr. Wisdom's usage I should have to utter to myself between each tick " Herenow is red " (cf. 80, 3), and say to others " I am seeing a red sense-datum which changes every second into an exactly similar sense-datum " (cf. 80, 10).

I do not wish to hold that such a way of talking about one's immediate visual experiences is " wrong ", but merely that it is only one way among others and has at least the one disadvantage of being likely to lead to confusions about change. In the situation just described we should ordinarily be inclined to say such a thing as " There has been *no change* in my visual field during the last ten seconds ". Thus there will be danger of confusion between this more ordinary use of the word " change ", and the curious one which Mr. Wisdom wishes to introduce. It will possibly be claimed that " a new kind of change has been discovered ".

The third " interesting conclusion " is that there are no unperceived sense-data (80, 19). The phrases " disbelief in unperceived sensibilia " (80, 22) and " (my view does not go) back to unperceived sensibilia " (80, 25) suggest that what has been discovered is that there are in the world no objects of a kind which can easily be described as those which are exactly like sense-data in every respect except that of being perceived. Actually it is nothing more than a misleading way of stating that such phrases as " An unperceived sense-datum continued to exist . . ." do not occur in Mr. Wisdom's terminology, i.e., that to use some such phrase is to say nothing at all unless you are using the word " sense-datum " in some way other than that in which Mr. Wisdom is using it. This follows from things he has already said about his use of the word. For in his use to say something about a sense-datum is to say that a colour is in a place which is being referred to by using the word " here " in a certain way. The word " here " is (by definition) being used in this way only if the person who is using it is seeing the place he is referring to. But propositions in whose expression the words " unperceived sense-datum " occur (in whatever use Mr. Wisdom has in mind) do not entail that a place in visual space is occupied by a colour

and that someone is seeing that place. While he is justified in saying that sentences containing the words " unperceived sense-data " are excluded by the rules of his terminology, he is not justified in saying that it is " incorrect " to speak of them, or that no terminology in which sense was given to the words " unperceived sense-datum " could usefully be employed to describe the same experience as he uses his terminology to describe.

I shall now try to sum up briefly what I have to say about Mr. Wisdom's difficulties about sense-data and his attempts to get rid of them.

The attempt to analyse sense-data, which gives the title to his paper, is merely incidental to the process of trying to clear up his " real " difficulties which he might be inclined to express somewhat as follows :—(1) Can one and the same sense-datum continue to exist throughout a period of time ? or are sense-data really only momentary ? and is what we are inclined to describe as " the same sense-datum continuing to exist throughout a period " *really* an infinite succession of momentary sense-data ? (2) Can sense-data change ? If so, when one sense-datum changes into another must it be always into a qualitatively different one ? can't it change into another the same ? (3) Mustn't all sense-data be perceived ?

He hopes to be able to answer these difficulties by trying as it were to dissect sense-data to find out what they are really like inside. In dissecting he comes upon a " core " (the Herenow), and by watching it closely he seems to be able to observe that it " can " endure throughout any period of time, that under such and such conditions it " can " change in a curious way, and that it " cannot " exist unperceived. From this he claims to deduce that *sense-data* " can " continue to exist throughout any period of time, " can " change in a peculiar way, and that there " can " be no unperceived ones.

But when we examine this process of " analysing sense-data " we see that it is merely the introducing of a terminology, partly old and partly new, to describe experiences of a certain sort.

The " conclusions " about identity, change, and perceptibility are not things that have been observed by seeing better or further into the structure of sense-data, but are ways of explaining what

that terminology is. To say that a "sense-datum *can* be numerically the same for a considerable time" (80, 8) is merely to say that in the terminology meaning is given to such a phrase as "a sense-datum has been numerically the same for a considerable time", the conditions under which it is to be correct to use the phrase being indicated (in very broad outline). To say that a sense-datum "*can* be changed into an exactly similar sense-datum" is to say that a meaning is to be given to such a phrase as "one sense-datum has been changed into another exactly similar sense-datum". These two "conclusions" tell us a way in which the word "sense-datum" is going to be used: the third one tells us a way in which it is *not* going to be used, viz., that no phrase such as "An unperceived sense-datum existed . . ." is going to be used.

But if someone prefers to use the word in some other way, it is not the case that he is "wrong": it is only the case that he is using it in some other way.

THE TRUTH OF RELIGIOUS PROPOSITIONS

By KARL BRITTON

IN *Analysis* for March 1934, Mr. C. A. Mace asked : " Is there not, so to speak, a syntax of expressive language—a duplicate of the syntax of simple expression ? " I am going to discuss this question—or rather, part of it : for I shall confine myself to language which expresses religious propositions. I want to contrast the meanings of the words " meaning," " truth," " syntax," as these apply respectively to scientific language and to the language of religious propositions.

By a sentence of scientific language, I mean one which is used to assert a perfectly precise meaning, and a meaning which must be true or false and can't be both. Such a sentence may mean a basic proposition or a hypothesis. A hypothesis-sentence can be translated according to invariable linguistic rules into basic-sentences. Basic sentences must include (besides logical constants) only signs for components which can be ostensively defined, and signs for constituents which can be ostensively denoted. The hypothesis is a truth-function of the basic propositions it implies. The truth or falsity of the basic proposition is determined by comparison with facts. Thus the *meaning* of a scientific sentence is the *showable* components and constituents to which it refers ; the *truth* of a scientific proposition is determined by the showable facts ; the *syntax* of scientific propositions all depends upon the law of excluded middle.

Philosophers have often regarded religious propositions as being scientific hypotheses. E.g. The Argument from Design to prove God's existence. But it has often been shown that, as such, they are *worthless* hypotheses. But it would be precarious to argue that religious propositions are altogether worthless.

Some people seem to me to *accept* religious propositions as being probable hypotheses, and yet to fail to grasp the real importance of religious propositions. To regard " God created the world " as a piece of information, a statement out of natural history, seems to me to be ignoring the religious proposition which these words may be used to convey. It is surely utterly repugnant to the religious mind to regard God's nature and

existence as doubtful, or even debatable.[1] And the actual truth of what look like scientific or historical propositions, in creeds and religious writings, is always quite irrelevent to the religious value of these propositions. The words "suffered under Pontius Pilate" are very like words which occur in Tacitus: but surely they are used for very different purposes. Again, such a sentence as "The Lord is my shepherd: I shall not want" seems to assert a proposition in economics. Yet a religious believer will not need economic evidence for the truth of this proposition, nor will he accept economic evidence of its falsity. We can only conclude that the words "I shall not want" are not here being used to assert a scientific proposition.

I am sure that many religious people think that sentences such as these do mean scientific hypotheses—whether established or probable. I have heard, for example, that a certain bishop proposed a chemical test to refute Transubstantiation. The horror of his opponents seems to have been due to superstition rather than to logical good taste. And I am sure that many people think that religion ends for a man who decides that religious propositions are *not* well-founded scientific hypotheses. In this I am sure they are mistaken.

On the other hand, religious people often insist that scientific criticism of religion misses the point. It does miss the point: but only because religious propositions cannot properly be called "true" or "false" in the sense proper to scientific propositions.

Recent writers who have adopted this view, have called religious sentences *meaningless*. But all now admit that they can be proved meaningless only in the sense of "meaning" proper to scientific sentences: but they *may* have "meaning" in some other sense. Of course some of them may be quite meaningless, in other senses too. Professor Broad is fond of referring to Dr. Tennant's words on Jacob Boehme[2]:

> "The critic is not given occasion to hesitate, by revelations that nothing can do something and at the same time not do it, or that a finite ego can at the same time be and not be: and he does well to call nonsense, by its name."
>
> (Philosophical Theology, I, 321.)

[1] Similarly I should hold that the existence of the material world may be certain, or may be a nonsensical notion: but surely it cannot be doubtful or only probable.

[2] Cf. a similar quotation from Heidegger, *Analysis*, May 1934. I am indebted to the recent articles in this paper by Mr. Ayer, Mr. Mace, Mr. W. H. F. Barnes, and Mr. G. R. T. Ross.

But no one can deny that contradictions sometimes have their uses. (e.g. Paradox) : the difficulty is to find an exact test for " meaningful " in any or all of its nonscientific senses.

One peculiarity of religious sentences is that they include signs for components and constituents which cannot be shown. We cannot ostensively denote *God*, nor define *Grace*. But this does not distinguish religious sentences from others, which are certainly used to assert scientific propositions : e.g. " I have a *pain* in my head ", " My *desire* is to come to you". We must further define religious sentences as those which mean propositions subject to a peculiar criterion for " truth " and " falsity ".

I think it is clear that anyone who asserts " The Lord is my shepherd " is usually willing to assert something about his own inner state—e.g. " I am feeling resigned and confident". Some religious people would even say that the *evidence* for the statement about God is to be found in facts about the inner lives of believers. (They would not in practice accept facts about the *outer* life of believers, as evidence for the truth or falsity of " The Lord is my Shepherd : " not, e.g. facts about the economic circumstances of believers.) Can we say, then, that the sentence " The Lord is my shepherd " (which I will call " R ") is used to make a statement about the psychology of believers ? Is " R " used for the indirect representation of such propositions as are more or less directly represented by the words : " I am feeling resigned and confident " ?

This would seem to show that " R " is scientific language only of an unscientific kind. For statements about psychology are meant as scientific propositions, and they might be translated into basic sentences (e.g. " I have a pain in my head " might be translated into basic sentences about my behaviour.)

But is " R " really used for the indirect representation of psychological propositions ? It is no less true that a person who asserts P (any scientific proposition) is willing to assert some psychological propositions about himself, e.g. " I believe P and intend to act on it". But surely it is not the case that P *implies* I believe P and shall act on it.[3]

Again, I am afraid that the peculiarity of *indirect* representation is that there are no invariable rules for translating from the

[3] Mr. Mace (op. cit.) says that " P " *expresses* belief in P, but *represents* P, and " no sentence can both represent and express the same fact."

language of indirect representation to the language of direct representation. A religious proposition is a work of art : it bears no rule-of-thumb relation to any scientific proposition.

And even if I were to make the maximum admission, then although " R " would be used to assert indirectly a scientific proposition (true or false in the straightforward sense), yet " R " may also be used for other purposes : it may have other meanings.

My view is that we cannot give an adequate account of religious propositions so long as we suppose that they do belong, or ought to belong, to the calculus of scientific propositions. This view involves that there are no religious facts, in the sense in which there are facts about physical objects ; that religious propositions are not true or false, in the ordinary sense of these words. My conclusion seems to follow from the view that religious propositions ought not to be submitted to scientific analysis and scientific rules of evidence—a view which many other religious people share. My conclusion might be contrasted with that of William James (Varieties of Religious Experience, p. 518).

But to suggest novel senses of " truth ", " meaning " and " syntax " is *not* to suggest that religious propositions might have a place in a *truth-value calculus* not deducible from the two-value Aristotelian logic. Surely part of the meaning of " true ", " false ", " meaning " as these apply to scientific language, is that every sentence which has meaning must be true or false and can't be both. Any system of Foundational Logic (or Theory of Deduction) must be based on some such law as the Law of Excluded Middle.[4] Any proposition which pretends to convey information must be subjected to the criticism of this logic. My suggestion is that religious propositions do not convey information : that they may however be "true" or " false " in a pickwickian sense. If we can give an exact definition of the pickwickian sense of " true", " false " and of " meaning", we shall then have precise tests of " truth " and of "meaning". This will give us a simple grammar, or set of rules, for the identification and certification of religious propositions. These rules are certainly not a system for Foundational Logic : they are not a

[4] Both the argument and the terminology I owe to Professor H. M. Sheffer of Harvard University, cf. C. I. Lewis and C. H. Langford : *Symbolic Logic*. Cap. vii.

truth-value system in the scientific sense of "truth". They might be compared to a calculus of Belief or a calculus for Euclidean Geometry.

Statements about the "truth" (or "R-validity") of a religious proposition, *would* purport to convey precise information, and to be *true* in the scientific sense of "truth". And the definitions of "true" and "meaning" would be rules of language, telling us how we ought to use the words in their novel senses. Both the statements and the rules, therefore, ought to be expressed in language which has meaning in the scientific sense of the word "meaning"—language which can be analysed according to the rules proper to scientific language.

What can we say of the uses of such a sentence as "The Lord is my Shepherd"? Has it a "meaning" or a "truth"-value?

Undoubtedly "R" may be used to express a mood, a feeling, the emotional tone of an experience or of a life-time. That is to say, "R" may be used to relieve a feeling or to convey a feeling. We may have recourse to any bodily activity to relieve our feelings: Mr. Mace questions whether words so used are really *language* at all. I think it is possible to distinguish between undiscriminating sound-making, and the discriminating use of sentences—both for the relief of emotion. It would seem to be the characteristic of a poet that his most important outlet for feeling lies in the use of certain chosen words. Similarly we all use language (with discrimination) to arouse emotion in others; but the *art* of doing so, is the art of poetry. There can be no rule-of-thumb.[5]

If "R" is used to express feelings, what can be the criterion of the validity of R? Religious people would (I think) vindicate their doctrines by an appeal to experience ("Inner Light") or to the testimony[6] of history, or to some definite human authority, or to any two, or all, of these, jointly.

No doubt there are differences of opinion about what is meant by "truth" of religious propositions. Theological discussions about Authority are really attempts to define the appropriate sense of "truth." Where different religious groups employ

[5] cf. Mr. F. R. Leavis' *New Bearings in English Poetry*.

[6] Both Testimony and Authority raise the question of "the common reference required or communication." What is the analysis of the sentence: "A understands B to assert that R"? See Miss Stebbing's remarks at Cardiff, as reported in *Analysis*, Oct. 1934, p. 25.

tests of a different type, they really use " truth " in different senses—although the differences may be inconsiderable.

I would suggest : R is true if the feelings and actions which " R " evokes in me (and perhaps also in a certain group of people), are accompanied in me (and perhaps in the same group of people, or perhaps in a different group—an " authority ") by feelings of moral approval.

This tentative account seems to me to assert a very important connexion between religious " truth " and ethical " truth "—the connexion implied in the familiar saying : " By their fruits ye shall know them".[1] There seems to be a widespread feeling that religious doctrines are valid if the feelings and actions which are characteristic of their believers, are morally good : but if the results in practice of believing R are bad, then R is false.

This view seems to embody an implicit belief that the whole point and purpose of religious propositions is to promote " good works " and a " state of Grace." It is certainly most repugnant to the religious mind to hold that the connexion of religion and morality is contingent and variable.

According to my view, we (or the proper authority) can estimate the " truth " or " falsity " of a religious doctrine by its psychological effects on those who accept it. We judge such effects by observing the behaviour of the believers. But this is *not* to say that a mere account of such behaviour is in itself any test of the truth of the beliefs. We must add to the account of the behaviour, an ethical or moral judgment of that behaviour.

But what can we say of the meaning and truth of ethical judgments ? It seems clear that such sentences as " This conduct is right " and " This man is good " are used to express feelings—feelings of approval, to attach a vague description. I incline to the view that they are also used to express the belief that my feelings are shared by some group or other. In some societies this group may be well defined. (The Roman Church has long had a group of men to whose moral feelings, questions of truth or falsity of doctrine have been submitted. Cf. Mr. T. S. Eliot's " After Strange Gods", on the character of orthodoxy.)

I hold a pragmatic view of the nature of religious truth.

[1] This suggestion I owe to Mr. J. F. Butler. See his article in the *Congregational Quarterly* for April 1935. " True " for Mr. Butler means true in the scientific sense.

" R is true " says something about the consequences of believing R—that they are good, and hence that it is useful to believe in R. But " good " and " useful " have to be defined in relation to a certain group of people, either all believers, or a priesthood of believers. It is their moral feelings that determine the meaning of " useful". It follows that religious truth is relative to a given group. The membership of such groups will be vague : there will be conversions from one group to another. A conversion is not a change of opinion, but a change of heart.

All that I have said about the test for truth, can be said in behaviouristic language. I could talk about the conduct of groups of people in relation to certain words and certain situations. These are my scientific propositions about " truth " in a nonscientific sense. Similarly, I might say that " R " has meaning when it is associated with certain kinds of behaviour : the behaviour may be good *or bad*, but it must be the sort of behaviour that can properly be styled good or bad.

In this way, we can make scientific statements about religious propositions, and suggest vaguely under what conditions " R " means a religious proposition, and under what conditions R is true. My results might be compared with C. S. Peirce's invaluable essay about " The Method of Tenacity", "The Method of Authority", "The Method of Reason", and " The Scientific Method " of discovering (or rather *defining*) Truth. (" Chance, Love and Logic " pp. 22–30.)

Aberystwyth, May 1935.

THE CRITERION OF TRUTH

By A. J. AYER

IN opposition to the physicalists, Professor Schlick maintains that the truth of a system of synthetic propositions does not consist merely in the freedom of the system from self-contradiction, but rather in its agreement with reality. Is there any sense in which we can accept this? I propose in this article to show that there is. And if we do accept this, have we then any ground for asserting, as Schlick does, that there is a class of synthetic propositions which are absolutely indubitable and thereby form the completely solid basis of all our empirical knowledge? I propose to show that we have not.

The first point that I want to establish is one that Ramsey expressed by saying that "there is really no separate problem of truth but merely a linguistic muddle".[1] For if we consider any sentence in which there is a reference to truth, we invariably find either that it is itself of the form '*p* is true', or else that it contains a sub-sentence of the form '*p* is true', and that when it is translated in such a way as to make this sub-sentence explicit it contains no other mention of truth. This the sentences 'all the propositions in Ramsey's article are true' and 'he has discovered a new truth', to take two examples at random, are equivalent respectively to "for all *p*, if *p* occurs in Ramsey's article, then *p* is true" and "there is a *p* such that he was the first to discover *p* and *p* is true". This indicates that to ask What is truth? is tantamount to asking What is the analysis of the sentence "*p* is true"? where the values of *p* are propositions. But it is evident that in a sentence of the form "*p* is true" or "it is true that *p*" the reference to truth never adds anything to the sense. If I say that it is true that Shakespeare wrote *Hamlet*, or that the proposition "Shakespeare wrote *Hamlet*" is true, I am saying no more than that Shakespeare wrote *Hamlet*. Similarly, if I say that it is false that Shakespeare wrote the *Iliad*, I am saying no more than that Shakespeare did not write the *Iliad*. And this shows that the words 'true' and 'false' are not used to stand for anything, but function in the sentence merely as assertion and negation signs. That is to say, *truth* and *falsehood*

[1] Foundations of Mathematics: Facts and Propositions, pp. 142–3.

are not genuine concepts. Consequently, there can be no logical problem concerning the nature of truth.[2]

Must we then regard all 'theories of truth' as attempts to answer a fictitious question, and so as nonsense? No, we may regard them as attempts to answer the genuine empirical question How are propositions actually validated? So that when Professor Schlick says that the truth of a synthetic proposition consists in its agreement with reality, and not merely in its logical compatibility with other propositions, we may assume that he is attempting to show what is the criterion that people actually apply to their beliefs. In this view, what he is saying is that, as a matter of fact, we are not prepared to accept a system of synthetic propositions as valid merely on the ground of its logical self-consistency, but require also that it should agree with reality. Can we accept this as a correct account of our procedure? I think we can, if we give a suitable interpretation to the objectionably vague expression 'agreement with reality'.

To begin with, we must make it clear that when we say that people test the validity of synthetic propositions by seeing if they agree with reality, we do not mean that they test their validity by comparing them, as physical objects, with other physical objects. I do not say that such a comparison is not legitimate, though I myself should distinguish propositions from the sentences which express them, and speak only of the sentences as physical objects; but it is surely irrelevant to the subject under discussion. The fact that it is significant to say, for example, that there are more words in a given sentence than there are chairs in a given room, has no connection that I can see with the question How are propositions validated? Yet it is only in this sense that propositions can be said to be literally comparable with reality. Accordingly, if we are to say that the condition which we expect synthetic propositions to satisfy is that of agreement with reality, we must make it clear that by 'agreement' we do not mean '*resemblance*'.

The word 'reality' is also misleading. We may concede to Dr. Hempel that it is often used in a metaphysical way, and had therefore better be avoided. But in the usage with which we are now concerned, it is not metaphysical. For it refers to nothing

[2] This argument is set forth at greater length in ch. 5 of my 'Language, Truth, and Logic.'

more than our own sensations. And when we say of a proposition that it 'agrees with reality' all that we mean, in this usage, is that our sensations are what the proposition in question said they would be. For every synthetic proposition may be regarded as asserting that certain sensations would occur in a given set of circumstances. And if, in a given case, the relevant conditions are fulfilled, and the sensations do occur, then, in Professor Schlick's terminology, the proposition is found to agree with reality. And we are taking it to be his contention that the criterion by which we all actually test the validity of synthetic propositions is their agreement with reality, in this sense.

As thus interpreted, Professor Schlick's view is surely intelligible, and I should say that it was correct. That is, I agree with him that people will accept a synthetic proposition as valid only if it conforms to their experience, in the sense just indicated, or they have reason to believe that it would do so. And I do not dispute what he says about our readiness to uphold the verdict of our own observations against the statements of other people, and about the feelings of pleasure which attend the verification of a hypothesis. But I think it should be emphasised that all these are contingent facts. There might very well be people who suffered great pain whenever their observations substantiated one of their hypotheses, and immediately abandoned it; and conversely there might be people who became extremely elated whenever their observations confuted one of their hypotheses, and immediately assumed it to have been substantiated. I do not say that such people exist; but I do say that their existence is not unthinkable. And therefore, while we may accept what Professor Schlick says as an accurate account of the procedure which we actually employ in connection with the validation of synthetic propositions, we must not assume that no other procedure is possible.

Unhappily, Professor Schlick in his defence of empiricism is not content with showing that the criterion by which we test the validity of empirical propositions is their accordance with our observations. He adds that the propositions which describe our immediate observations are absolutely indubitable. And here I am unable to follow him. For one thing, I find it very difficult to see what Professor Schlick and his followers really mean to assert when they say that propositions which record direct observations

are 'certain' or 'indubitable' or 'unalterable'. Presumably, they are not referring to our subjective attitude towards these propositions. They are not merely saying that we are absolutely confident of the truth of these propositions, directly they are formulated, and never bring ourselves to doubt them. At least if they do intend to assert this, it is not all that they intend to assert when they say that propositions recording direct observations are indubitable. They apparently mean also to assert that these propositions are *objectively* certain. But the only sense which I can attach to the statement that a given proposition is objectively certain is that it is such that its contradictory is self-contradictory. Yet if we consider propositions such as 'this is green' or 'I feel pain' which are given us as examples of propositions recording direct observations, we do not find that they are objectively certain in this sense. 'This is not green' is not self-contradictory. What does seem to me self-contradictory is to maintain that a synthetic proposition can be valid in virtue of its form alone. For this is surely the characteristic mark of an analytic proposition.

To this I suppose that Professor Schlick would reply that he never suggested that a proposition such as 'I am in pain' or 'this is green' was valid in virtue of its form alone, or that it could not legitimately be contradicted. All that he maintained, he would say, was that *at the moment at which I was actually having a sensation of pain, or of green* the proposition 'this is green' or 'I am in pain' which I might use to describe my immediate sensations would be absolutely 'objectively' certain. But here we must distinguish between two Propositions one of which is false and the other true, but trivial. It is true to say that it is 'objectively' certain that when I am feeling pain then I am indeed feeling pain. For this is to say no more than that it is objectively certain that *p* implies *p*. But it is false to say that when I am feeling pain, then the proposition that I am feeling pain is objectively certain. For this is a special case of the proposition '*p* implies that (*p* is objectively certain)'. And this proposition is false when, as is the case here, *p* is a synthetic proposition. I can not help thinking that Professor Schlick and his followers have been guilty of confusing these two propositions, and have come to believe the second only because they mistook it for the first.

That Professor Schlick is himself uneasy about his own view is

indicated by his vagueness concerning the nature of his 'Konstatierungen', which he supposes to be absolutely indubitable. He speaks of them as propositions, yet he says that they are not to be identified with anything that can be expressed, which suggests that he is thinking of the actual sensations, as opposed to the propositions which describe them. But here we may present him with a dilemma. If by 'Konstatierungen' he means actual sensations, then it is nonsensical to say that they either are or are not indubitable. For a sensation is not the sort of thing that can be doubtful, or not doubtful. It simply occurs. If on the other hand he means by 'Konstatierungen' the propositions which we use to describe our sensations, then we have shown that these propositions are not 'objectively certain', in the only sense which we are able to attach to this expression.